Now $AP + PR + RD$, or $PA + (n \div m).PB + (r \div m).PC$ will be least when $APRD$ becomes a straight line. This will be the case when P is taken so that the angle APB is the supplement of BPR, and BPC or BRD is the supplement of BRP. Hence we have the following

Construction. – On the sides BC, CA, AB construct the triangles BCD, CAE, ABF, making them similar to BPR, the angle $CBD = ABF = CEA = PBR$, $BCD = ACE = AFB = BPR$, and $CAE = BAF = BDC = BRP$. Circumscribe circles about the trian's BCD, CAE, ABF.

Now since $BDC + CEA + AFB = 180°$, the sum of the angles inscribed in the arcs BC, CA, AB, is 360°; hence the circumferences intersect in one point O, which is the required point. Join OD, OE, OF, and make the angle $OBH = CBD$.

The angle BOD is equal to BCD, which by construction is equal BFA, and BFA is the supplement of AOB; hence BOD is the supplement of AOB, and AOD is, therefore, a straight line. Similarly it can be proved that BOE and COF are straight lines.

The triangle BOH being similar to BCD, and BHD to BOC, we have $OH = (n \div m).OB$, and $HD = (r \div m).OC$; $\therefore AD = OA + (n \div m).OB + (r \div m).OC$, and $m.AD = n.OA + m.OB + r.OC =$ the req'd minimum.

In like manner we prove that $n.BE$ and $r.CF$ each = the req'd min.

If Δ, Δ_1 are the areas of the $\triangle ABC$ and a $\triangle$ with sides m, n, r, and if α, β, γ represent the angles A, B, F, of the $\triangle ABF$, we find,

$$OA = \frac{2m\Delta(\cot A + \cot \alpha)}{\sqrt{[2\Delta_1(a^2\cot\alpha + b^2\cot\beta + c^2\cot\gamma + 4\Delta)]}},$$

$$OB = \frac{2n\Delta(\cot B + \cot \beta)}{\sqrt{[2\Delta_1(a^2\cot\alpha + b^2\cot\beta + c^2\cot\gamma + 4\Delta)]}},$$

$$OC = \frac{2r\Delta(\cot C + \cot \gamma)}{\sqrt{[2\Delta_1(a^2\cot\alpha + b^2\cot\beta + c^2\cot\gamma + 4\Delta)]}}.$$

PROBLEMS.

259. By Prof. J. Scheffer, *Mercersberg, Pa.*—In a triangle, one side = 400 ft., and the two adjacent angles, 70° and 80°, are given; to compute the other two sides without the aid of trigonometry.

260. By Geo. M. Day, *Lockport, N. Y.*—A sphere, radius r, rolls down the concave arc of a circle, radius R. At the beginning of the motion, the center of the sphere is on the horizontal diameter of the circle. Find the time of descent of the sphere in terms of the coordinable of its center.

261. By ALEX. S. CHRISTIE, *U. S. Coast Survey, Washington, D. C.*—A curve of the nth degree rolls upon any curve whatever: to determine the degree and position of the locus of the centers of curvature of all the elements described, at any given instant, by the points of the rolling curve.

262. By PROF. A. B. EVANS, *Lockport, N. Y.*—Prove, x being an integer, that $x(x+1)(x+2)(x+3)(x+4)(x+5)(x+6)$ cannot be the square of a commensurate number.

263. By W. L. MARCY, *Colorado Springs, Col.*—On the 17th of Aug., 1878, at $5^h\ 20^m$ A. M., an observation of polaris was taken in Lat. 37°30′, Long. 107° W. from Greenwich, with an engineer's transit; from the meridian thus obtained, the Sun's azimuth at $6^h 23^m 53^s$, A. M., was 79°10′30″

What was the error of the watch, and of the meridian?

264. By PROF. J. H. KERSHNER, *Mercersberg, Pa.*—It is required to divide a given straight line into three parts such that the triangle formed of them shall have its circumscribing circle a minimum and its inscribed circle a maximum.

265. By GEORGE EASTWOOD, *Saxonville, Mass.*—At age a, a person takes out a life policy of k dollars, for which he agrees to pay an annual premium of p dollars. At age $a+n$ he is not able to make his annual payment and wants to sell, or surrender, his life-policy for full-paid insurance policy. If the n premiums that have been paid be each viewed as the sum of a series of infinitely small annuities, payable at infinitely small intervals of time, within each year, what ought to be the present value of the new policy?

PUBLICATIONS RECEIVED.

Transactions of the Wisconsin Academy of Science, Arts and Letters. Madison, Wisconsin. 1879. 320 pages, 8vo.

The Mathematical Visitor, No. 3. Artemas Martin, A. M., Editor and Publisher. Erie, Pa. 36 pages, 4to. Price, 50 cents.

ERRATA.

On page 63, line 20, for Prof. Johnson, read, Prof. Hendrickson.
" " 67, " 9, insert a period after $\varpi+1$, and for "Le", read Let.
" " 49, " 4, Vol. III, for 4058, read 4059, and for 4059, read, 4060.
" " 23, Vol. IV, insert the factor, $\cos\phi$, in the numerator of the value of $\triangle$.